Math Class 11 Chapter 1 and 2 Sets and relations and functions

Set

Set is a collection of well defined objects which are distinct from each other. Sets are usually denoted by capital letters A, B,C,… and elements are usually denoted by small letters a, b,c,… .

If 'a' is an element of a set A, then we write $a \in A$ and say 'a' belongs to A or 'a' is in A or 'a' is a member of A. If 'a' does not belongs to A, we write $a \notin A$.

Standard Notations

- N : A set of natural numbers.
- W : A set of whole numbers.
- Z : A set of integers.
- Z^+/Z^- : A set of all positive/negative integers.
- Q : A set of all rational numbers.
- Q^+/Q^- : A set of all positive/ negative rational numbers.
- R : A set of real numbers.
- R^+/R^-: A set of all positive/negative real numbers.
- C : A set of all complex numbers.

Methods for Describing a Set

(i) Roster/Listing Method/Tabular Form In this method, a set is described by listing element, separated by commas, within braces.
e.g., A = {a, e, i, o, u}

(ii) **Set Builder/Rule Method** In this method, we write down a property or rule which gives us all the elements of the set by that rule.
e.g.,A = {x : x is a vowel of English alphabets}

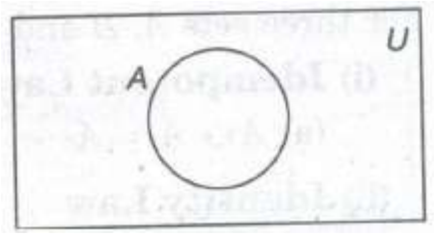
Types of Sets

1. **Finite Set** A set containing finite number of elements or no element.
2. **Cardinal Number of a Finite Set** The number of elements in a given finite set is called cardinal number of finite set, denoted by n (A).
3. **Infinite Set** A set containing infinite number of elements.
4. **Empty/Null/Void Set** A set containing no element, it is denoted by (φ) or { }.
5. **Singleton Set** A set containing a single element.
6. **Equal Sets** Two sets A and B are said to be equal, if every element of A is a member of B and every element of B is a member of A and we write A = B.

7. **Equivalent Sets** Two sets are said to be equivalent, if they have same number of elements.
   If n(A) = n(B), then A and B are equivalent sets. But converse is not true.
8. **Subset and Superset** Let A and B be two sets. If every element of A is an element of B, then A is called subset of B and B is called superset of A. Written as
   $A \subseteq B$ or $B \supseteq A$
9. **Proper Subset** If A is a subset of B and $A \neq B$, then A is called proper subset of B and we write $A \subset B$.
10. **Universal Set** (U) A set consisting of all possible elements which occurs under consideration is called a universal set.
11. **Comparable Sets** Two sets A and Bare comparable, if $A \subseteq B$ or $B \subseteq A$.
12. **Non-Comparable Sets** For two sets A and B, if neither $A \subseteq B$ nor $B \subseteq A$, then A and B are called non-comparable sets.
13. **Power Set** (P) The set formed by all the subsets of a given set A, is called power set of A, denoted by P(A).
14. **Disjoint Sets** Two sets A and B are called disjoint, if, $A \cap B = (\varphi)$.

Venn Diagram

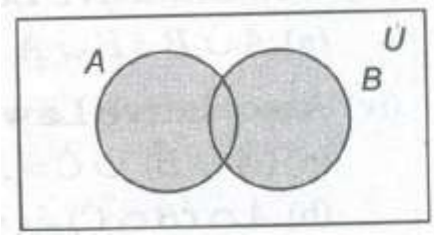
In a Venn diagram, the universal set is represented by a rectangular region and a set is represented by circle or a closed geometrical figure inside the universal set.



Operations on Sets

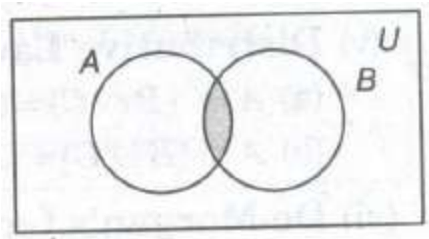
1. Union of Sets

The union of two sets A and B, denoted by $A \cup B$ is the set of all those elements, each one of which is either in A or in B or both in A and B.



2. Intersection of Sets

The intersection of two sets A and B, denoted by A ∩ B, is the set of all those elements which are common to both A and B.

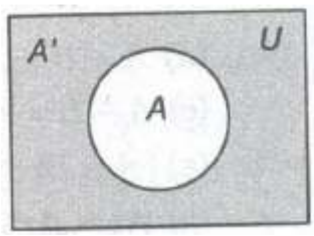


If $A_1, A_2, \ldots, A_n$ is a finite family of sets, then their intersection is denoted by

$$\bigcap_{i=1}^{n} A_i \text{ or } A_1 \cap A_2 \cap \ldots \cap A_n$$

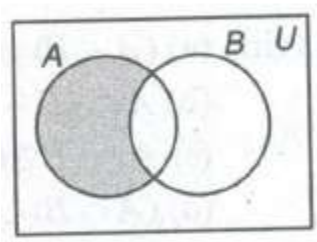
3. Complement of a Set

If A is a set with U as universal set, then complement of a set, denoted by A' or A^c is the set U – A .



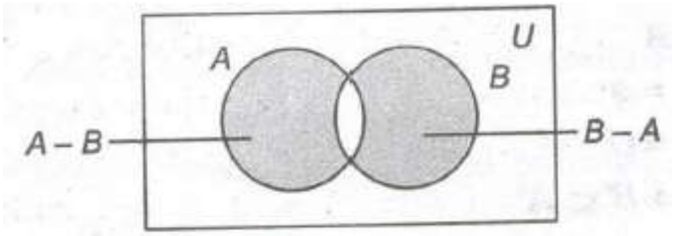
4. Difference of Sets

For two sets A and B, the difference A – B is the set of all those elements of A which do not belong to B.



5. Symmetric Difference

For two sets A and B, symmetric difference is the set (A – B) ∪ (B – A) denoted by A Δ B.



Laws of Algebra of Sets

For three sets A, B and C

(i) Commutative Laws

$A \cap B = B \cap A$
$A \cup B = B \cup A$

(ii) Associative Laws

$(A \cap B) \cap C = A \cap (B \cap C)$
$(A \cup B) \cup C = A \cup (B \cup C)$

(iii) Distributive Laws

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(iv) Idempotent Laws

$A \cap A = A$
$A \cup A = A$

(v) Identity Laws

$A \cup \Phi = A$
$A \cap \mathbf{U} = A$

(vi) De Morgan's Laws

(a) $(A \cap B)' = A' \cup B'$
(b) $(A \cup B)' = A' \cap B'$
(c) $A - (B \cap C) = (A - B) \cap (A - C)$
(d) $A - (B \cup C) = (A - B) \cup (A - C)$

(vii) (a) $A - B = A \cap B'$
(b) $B - A = B \cap A'$
(c) $A - B = A \Leftrightarrow A \cap B = (\Phi)$

(d) $(A - B) \cup B = A \cup B$
(e) $(A - B) \cap B = (\Phi)$
(f) $A \cap B \subseteq A$ and $A \cap B \subseteq B$
(g) $A \cup (A \cap B) = A$
(h) $A \cap (A \cup B) = A$

(viii) (a) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$
(b) $A \cap (B - C) = (A \cap B) - (A \cap C)$
(c) $A \cap (B \Delta C) = (A \cap B)$ A $(A \cap C)$
(d) $(A \cap B) \cup (A - B) = A$
(e) $A \cup (B - A) = (A \cup B)$

(ix) (a) $U' = (\Phi)$
(b) $\Phi' = U$
(c) $(A')' = A$
(d) $A \cap A' = (\Phi)$
(e) $A \cup A' = U$
(f) $A \subseteq B \Leftrightarrow B' \subseteq A'$

Important Points to be Remembered

- Every set is a subset of itself i.e., $A \subseteq A$, for any set A.
- Empty set Φ is a subset of every set i.e., $\Phi \subset A$, for any set A.
- For any set A and its universal set U, $A \subseteq U$
- If $A = \Phi$, then power set has only one element i.e., $n(P(A)) = 1$
- Power set of any set is always a non-empty set.
  Suppose $A = \{1, 2\}$, thenP(A) = $\{\{1\}, \{2\}, \{1, 2\}, \Phi\}$.(a) $A \notin P(A)$
  (b) $\{A\} \in P(A)$
- (vii) If a set A has n elements, then P(A) or subset of A has 2^n elements.
- (viii) Equal sets are always equivalent but equivalent sets may not be equal.

The set $\{\Phi\}$ is not a null set. It is a set containing one element Φ.

Results on Number of Elements in Sets

- $n(A \cup B) = n(A) + (B) - n(A \cap B)$
- $n(A \cup B) = n(A) + n(B)$, if A and B are disjoint.
- $n(A - B) = n(A) - n(A \cap B)$
- $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
- n (number of elements in exactly two of the sets A, B, C) $= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$
- n (number of elements in exactly one of the sets A, B, C) $= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$
- $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$

- $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$
- $n(B - A) = n(B) - n(A \cap B)$

Ordered Pair

An ordered pair consists of two objects or elements in a given fixed order.

Equality of Ordered Pairs Two ordered pairs (a_1, b_1) and (a_2, b_2) are equal iff $a_1 = a_2$ and $b_1 = b_2$.

Cartesian Product of Sets

For two sets A and B (non-empty sets), the set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$ is called Cartesian product of the sets A and' B, denoted by A x B.

$A \times B=\{(a,b):a \in A \text{ and } b \in B\}$

If there are three sets A, B, C and $a \in A$, be B and $c \in C$, then we form, an ordered triplet (a, b, c). The set of all ordered triplets (a, b, c) is called the cartesian product of these sets A, B and C.

i.e., $A \times B \times C = \{(a,b,c):a \in A,b \in B,c \in C\}$

Properties of Cartesian Product

For three sets A, B and C

- $n(A \times B)= n(A)\, n(B)$
- $A \times B = \Phi$, if either A or B is an empty set.
- $A \times (B \cup C)= (A \times B) \cup (A \times C)$
- $A \times (B \cap C) = (A \times B) \cap (A \times C)$
- $A \times (B - C)= (A \times B) - (A \times C)$
- $(A \times B) \cap (C \times D)= (A \cap C) \times (B \cap D)$
- If $A \subseteq B$ and $C \subseteq D$, then $(A \times C) \subset (B \times D)$
- If $A \subseteq B$, then $A \times A \subseteq (A \times B) \cap (B \times A)$
- $A \times B = B \times A \Leftrightarrow A = B$
- If either A or B is an infinite set, then A x B is an infinite set.
- $A \times (B' \cup C')' = (A \times B) \cap (A \times C)$
- $A \times (B' \cap C')' = (A \times B) \cup (A \times C)$
- If A and B be any two non-empty sets having n elements in common, then A x B and B x A have n^2 elements in common.
- If $\neq B$, then $A \times B \neq B \times A$
- If $A = B$, then $A \times B= B \times A$
- If $A \subseteq B$, then $A \times C = B \times C$ for any set C.

Relation

If A and B are two non-empty sets, then a relation R from A to B is a subset of A x B.

If $R \subseteq A \times B$ and $(a, b) \in R$, then we say that a is related to b by the relation R, written as aRb.

Domain and Range of a Relation

Let R be a relation from a set A to set B. Then, set of all first components or coordinates of the ordered pairs belonging to R is called : the domain of R, while the set of all second components or coordinates = of the ordered pairs belonging to R is called the range of R.

Thus, domain of $R = \{a : (a , b) \in R\}$ and range of $R = \{b : (a, b) \in R\}$

Types of Relations

(i) Void Relation As $\Phi \subset A \times A$, for any set A, so Φ is a relation on A, called the empty or void relation.

(ii) Universal Relation Since, $A \times A \subseteq A \times A$, so A x A is a relation on A, called the universal relation.

(iii) Identity Relation The relation $I_A = \{(a, a) : a \in A\}$ is called the identity relation on A.

(iv) Reflexive Relation A relation R is said to be reflexive relation, if every element of A is related to itself.

Thus, $(a, a) \in R, \forall a \in A = R$ is reflexive.

(v) Symmetric Relation A relation R is said to be symmetric relation, iff

$(a, b) \in R$ $(b, a) \in R, \forall a, b \in A$

i.e., $a R b \Rightarrow b R a, \forall a, b \in A$

$\Rightarrow$ R is symmetric.

(vi) Anti-Symmetric Relation A relation R is said to be anti-symmetric relation, iff

$(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b, \forall a, b \in A$

(vii) Transitive Relation A relation R is said to be transitive relation, iff $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow (a, c) \in R, \forall a, b, c \in A$

(viii) Equivalence Relation A relation R is said to be an equivalence relation, if it is simultaneously reflexive, symmetric and transitive on A.

(ix) Partial Order Relation A relation R is said to be a partial order relation, if it is simultaneously reflexive, symmetric and anti-symmetric on A.

(x) Total Order Relation A relation R on a set A is said to be a total order relation on A, if R is a partial order relation on A.

Inverse Relation

If A and B are two non-empty sets and R be a relation from A to B, such that R = {(a, b) : a ∈ A, b ∈ B}, then the inverse of R, denoted by R^{-1} , i a relation from B to A and is defined by

$R^{-1} = \{(b, a) : (a, b) \in R\}$

Equivalence Classes of an Equivalence Relation

Let R be equivalence relation in A (≠ Φ). Let a ∈ A.

Then, the equivalence class of a denoted by [a] or {a} is defined as the set of all those points of A which are related to a under the relation R.

Composition of Relation

Let R and S be two relations from sets A to B and B to C respectively, then we can define relation SoR from A to C such that (a, c) ∈ So R ⇔ ∃ b ∈ B such that (a, b) ∈ R and (b, c) ∈ S.

This relation SoR is called the composition of R and S.

(i) RoS ≠ SoR
(ii) $(SoR)^{-1} = R^{-1}oS^{-1}$

known as **reversal rule.**

Congruence Modulo m

Let m be an arbitrary but fixed integer. Two integers a and b are said to be congruence modulo m, if a – b is divisible by m and we write a ≡ b (mod m).

i.e., a ≡ b (mod m) ⇔ a – b is divisible by m.

Important Results on Relation

- If R and S are two equivalence relations on a set A, then R ∩ S is also on ‘equivalence relation on A.

- The union of two equivalence relations on a set is not necessarily an equivalence relation on the set.
- If R is an equivalence relation on a set A, then R^{-1} is also an equivalence relation on A.
- If a set A has n elements, then number of reflexive relations from A to A is 2^{n2-2}
- Let A and B be two non-empty finite sets consisting of m and n elements, respectively. Then, A x B consists of mn ordered pairs. So, total number of relations from A to B is 2^{nm}.

Binary Operations

Closure Property

An operation * on a non-empty set S is said to satisfy the closure ‘ property, if

$a \in S, b \in S \Rightarrow a * b \in S, \forall a, b \in S$

Also, in this case we say that S is closed for *.

An operation * on a non-empty set S, satisfying the closure property is known as a binary operation.

or

Let S be a non-empty set. A function f from S x S to S is called a binary operation on S i.e., f : S x S → S is a binary operation on set S.

Properties

- Generally binary operations are represented by the symbols * , +, … etc., instead of letters figure etc.
- Addition is a binary operation on each one of the sets N, Z, Q, R and C of natural numbers, integers, rationals, real and complex numbers, respectively. While addition on the set S of all irrationals is not a binary operation.
- Multiplication is a binary operation on each one of the sets N, Z, Q, R and C of natural numbers, integers, rationals, real and complex numbers, respectively. While multiplication on the set S of all irrationals is not a binary operation.
- Subtraction is a binary operation on each one of the sets Z, Q, R and C of integers, rationals, real and complex numbers, respectively. While subtraction on the set of natural numbers is not a binary operation.
- Let S be a non-empty set and P(S) be its power set. Then, the union and intersection on P(S) is a binary operation.
- Division is not a binary operation on any of the sets N, Z, Q, R and C. However, it is not a binary operation on the sets of all non-zero rational (real or complex) numbers.
- Exponential operation $(a, b) \to a^b$ is a binary operation on set N of natural numbers while it is not a binary operation on set Z of integers.

Types of Binary Operations

(i) Associative Law A binary operation * on a non-empty set S is said to be associative, if (a * b) * c = a * (b * c), ∀ a, b, c ∈ S.

Let R be the set of real numbers, then addition and multiplication on R satisfies the associative law.

(ii) Commutative Law A binary operation * on a non-empty set S is said to be commutative, if

a * b = b * a, ∀ a, b ∈ S.

Addition and multiplication are commutative binary operations on Z but subtraction not a commutative binary operation, since

$2 - 3 \neq 3 - 2$.

Union and intersection are commutative binary operations on the power P(S) of all subsets of set S. But difference of sets is not a commutative binary operation on P(S).

(iii) Distributive Law Let * and o be two binary operations on a non-empty sets. We say that * is distributed over o., if

a * (b o c)= (a * b) o (a * c), ∀ a, b, c ∈ S also called (left distribution) and (b o c) * a = (b * a) o (c * a), ∀ a, b, c ∈ S also called (right distribution).

Let R be the set of all real numbers, then multiplication distributes addition on R.

Since, a.(b + c) = a.b + a.c,∀ a, b, c ∈ R.

(iv) Identity Element Let * be a binary operation on a non-empty set S. An element e a S, if it exist such that

a * e = e * a = a, ∀ a ∈ S.

is called an identity elements of S, with respect to *.

For addition on R, zero is the identity elements in R.

Since, a + 0 = 0 + a = a, ∀ a ∈ R

For multiplication on R, 1 is the identity element in R.

Since, a x 1 =1 x a = a,∀ a ∈ R

Let P (S) be the power set of a non-empty set S. Then, Φ is the identity element for union on P (S) as

$A \cup \Phi = \Phi \cup A = A, \forall A \in P(S)$

Also, S is the identity element for intersection on P(S).

Since, $A \cap S = A \cap S = A, \forall A \in P(S)$.

For addition on N the identity element does not exist. But for multiplication on N the idenity element is 1.

(v) Inverse of an Element Let * be a binary operation on a non-empty set ‘S’ and let ‘e’ be the identity element.

Let $a \in S$. we say that a^{-1} is invertible, if there exists an element $b \in S$ such that $a * b = b * a = e$

Also, in this case, b is called the inverse of a and we write, $a^{-1} = b$

Addition on N has no identity element and accordingly N has no invertible element.

Multiplication on N has 1 as the identity element and no element other than 1 is invertible.

Let S be a finite set containing n elements. Then, the total number of binary operations on S in n^{n2}

Let S be a finite set containing n elements. Then, the total number of commutative binary operation on S is n [n(n+1)/2].